Midterm 2 Study Problem answers

(20pts)

1. Factor each of the following polynomials into linear terms with integer coefficients:

$$h(x) = x^4 - x^3 - 7x^2 + x + 6$$

$$g(x) = x^3 - 3x^2 + 4$$

By using the rational roots theorem and base-x long division, you should be able to determine that:

$$h(x) = (x - 1)(x + 1)(x + 2)(x - 3)$$

and

$$g(x) = (x + 1)(x - 2)^2$$

■

Midterm 2 Study Problem answers

(25pts)

2. $f(x) = (x-1)(x+1)(x+2)(x-3)/(x+1)\,(x-2)(x-2)$

i) The x+1 terms cancel, so we have a hole at the point (-1, f(-1)).

ii) The roots, where the graph crosses the x-axis, are at x=1, -2 and 3. (in orange on graph)

iii) The vertical asymptote is the line x=2. (in blue)

iv) The y-intercept is at (0,1.5) (in orange)

v) Since the degree of h(x) is one greater than g(x), we have a slant asymptote. Since the degree is not equal, or the degree of g(x) is not greater than the degree of h(x), we do not have horizontal asymptotes. (you might on the test, be sure you know how to find them!).

vi) To find the slant asymptote, use base-x to divide the ORIGINAL g(x) into h(x) and throw away the remainder. If you do you should get the slant equation: y=x+2. Now we should check to see if the graph crosses the slant eq. (you should also do this for horizontal asymptotes!)

You can use the equations with the x+1 term canceled. By cross multiplying you end up with:

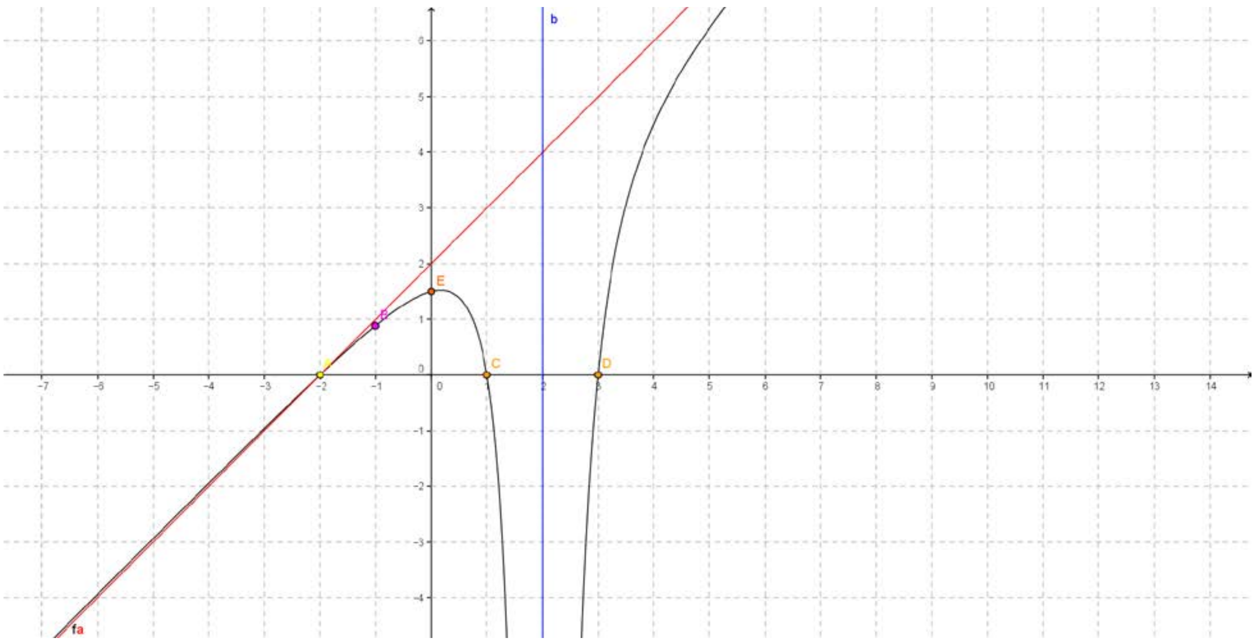
$$x^3 - 2x^2 - 5x + 6 = x^3 - 2x^2 - 4x + 8$$

Which has a lot of cancelling, and simplifies quickly to x=-2. This means the graph crosses the slant equation there, before heading off asymptotically towards the line as x goes to negative infinity. This point is in yellow on graph.

Midterm 2 Study Problem answers

By using sign analysis, plugging in particular points, the graph can be sketched easily as:

The hardest part is to compare y=x+2 to other points plugged in. if you plug in say -4, you will find the function is higher than plugging in to the slant equation so you know that when it crosses the slant, it is higher above it, but as x goes to –infinity, it will tend towards it.



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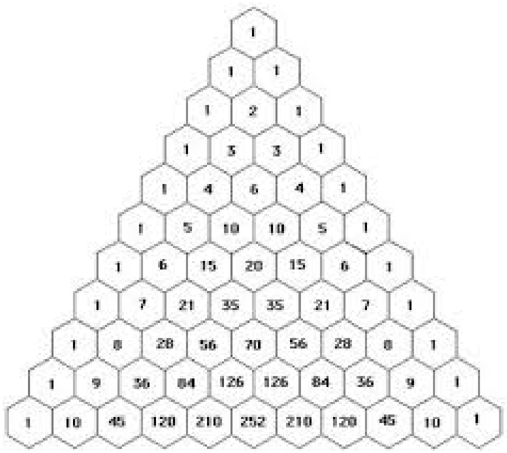
Midterm 2 Study Problem answers

(15pts)

3. Using your own derivation of Pascal's triangle, determine the following for the binomial expansion of:

$$\left(-\frac{x^3}{2}-\frac{4}{x^7}\right)^{10}$$



a. The middle term. $(252)\left(-\frac{x^3}{2}\right)^5\left(-\frac{4}{x^7}\right)^5=\frac{8064}{x^{20}}$

b. The 7th term. $(210)\left(-\frac{x^3}{2}\right)^4\left(-\frac{4}{x^7}\right)^6=\frac{53760}{x^{30}}$

c. The coefficient of the term with no x.

Given $\left(-\frac{x^3}{2}\right)^a\left(-\frac{4}{x^7}\right)^b$ one can find the equations: $a+b=10$ and $3a=7b$ which means that $3a-7b=0$. Solving this system of two equations finds that:

$a=7$ and $b=3$.

$$(120)\left(-\frac{x^3}{2}\right)^7\left(-\frac{4}{x^7}\right)^3=60 \ ■$$

Midterm 2 Study Problem answers

(20pts)

4. Find a polynomial with integer coefficients that has $1+\sqrt[3]{2}$ and $1+2i$ as roots.

First find polynomial with $1+\sqrt[3]{2}$ as a root. Let $x=1+\sqrt[3]{2}$ so:

$x-1=\sqrt[3]{2}$. Now cube both sides to get:

$x^3-3x^2+3x-1=2$. Then our first polynomial that we could use is:

$$f(x)=x^3-3x^2+3x-3$$

Now, like in the previous material, find a polynomial having $1+2i$

As a root. Since we want integer coefficients, we must realize that $1-2i$ is also a root.

So: $\big(x-(1+2i)\big)\big(x-(1-2i)\big)=(x-1-2i)(x-1+2i)=x^2-2x+5$

Now just use base-x to multiply the two polynomials you found, together to get:

$x^5-5x^4+14x^3-24x^2+21x-15$ as an answer. Multiplying this by anything non-zero, even involving x, would still be an answer.■

Midterm 2 Study Problem answers

(5pts)

5. Solve the following radical equation:

$$x - 3 = \sqrt{30 - 2x}$$

Squaring both sides gives:

$$x^2 - 6x + 9 = 30 - 2x$$

Setting everything to zero gives:

$$x^2 - 4x - 21 = 0 = (x - 7)(x + 3)$$

Now plugging in 7 or -3 we find that -3 gives the result that -6=6, which is false, so 7 is the only solution. Remember that $\sqrt{36} \neq \pm 6$. Don't confuse this with solving the EQUATION $x^2 = 36$ which has the solutions ± 6! ■

(5pts)

6. Solve the equation: $3(2^{x+4}) = 350$, rounding the answer to three decimal places.

Divide both sides by 3 to get:

$$2^{x+4} = 116.667$$

Now take the log base 2 of both sides, remembering you need the change of base formula:

$$log_2(2^{x+4}) = log_2(116.667) = \frac{\log(116.667)}{log2} = 6.866 = x + 4$$

$$x = 6.866 - 4 = 2.866$$■

Midterm 2 Study Problem answers

(5pts)

7. Given $ln2 = 0.693$ and $ln4 = 1.386$, then find the following ONLY using properties of logarithms and basic arithmetic (you must show your work!)
   a. $ln8 = ln(2 \cdot 4) = ln2 + ln4 = .693 + 1.386 = 2.079$
   b. $ln\sqrt{2} = ln2^{1/2} = \frac{1}{2}ln2 = (.5)(.693) = .3465$
   c. $ln\frac{1}{64} = ln4^{-3} = (-3)ln4 = (-3)(1.386) = -4.158$ ■

(5pts)

8. Write as a single simplified term without logarithms:

$$10^{6logxy+logy-3logx}$$

$$= 10^{log(xy)^6+logy-logx^3} = 10^{log\frac{x^6y^6y}{x^3}} = 10^{logx^3y^7} = x^3y^7$$